Comparison of ARIMA Parameter Estimation Using Maximum Likelihood and Bayesian With Gamma Distribution on IHSG Data

Muhamamd Fahmi Siloto[1*], Kartika Harahap[2], Sutarman[3]

[1,2] Student Faculty of Mathematics and Natural Sciences, Universitas Sumatera Utara, Medan, Indonesia
[3] Lecture Faculty of Mathematics and Natural Sciences, Universitas Sumatera Utara, Medan, Indonesia
*Coresponding Author:
Email: muhammadfahmisiloto@gmail.com

***Abstract**.*

This study aims to examine the differences in the parameter estimation results of the ARIMA(0,1,2) model with Gamma-distributed residuals using two statistical approaches, namely Maximum Likelihood (ML) and Bayesian. The ML estimation results show that the parameters MA(1) ≈ 0.0028 and MA(2) ≈ 0.0370 are very small, indicating that the influence of the previous residuals on daily JCI changes is relatively weak after the differencing process. The Gamma shape parameter of 3.7368 reflects residuals that tend to be symmetrical. The Neg Log-Likelihood value of 1350.98 produces an AIC of 2707.96 as an indicator of model fit. In contrast, the Bayesian approach provides estimates of MA(1) ≈ 0.25 and MA(2) ≈ 0.15 which are larger than ML, reflecting estimates that consider parameter uncertainty through the posterior distribution. The resulting Gamma parameters (α ≈ 3, β ≈ 0.015) show positive residual characteristics with moderate variations.

Keywords: *ARIMA; Maximum Likelihood; Bayesian; Gamma Distribution and IHSG Data.*

I. INTRODUCTION

Statistics plays a crucial role in modeling uncertain phenomena, particularly through the analysis of time series data. Time series data are characterized by the interdependence of observations over time, so the analytical method must be able to capture the dynamics and correlation structure of the data. One of the primary objectives of time series analysis is to construct a mathematical model that accurately represents data patterns and can be used for forecasting. The Autoregressive Integrated Moving Average (ARIMA) model is one of the most fundamental and widely used models in time series analysis. This model combines autoregressive (AR), differencing (I), and moving average (MA) components to handle non-stationary data and model linear dependencies between periods. Within the ARIMA framework, the stochastic process of data is modeled through parameters that represent the influence of past values and previous residuals on current values. The accuracy of the ARIMA model depends heavily on the parameter estimation method used.Parameter estimation aims to determine the model coefficient values that best fit the data, so the selection of the estimation method is a crucial aspect in statistical analysis. Two main approaches commonly used in parameter estimation are the Maximum Likelihood approach and the Bayesian approach. The Maximum Likelihood (ML) method is a classical approach that estimates parameters by maximizing the likelihood function, which is the probability of the observed data given a given parameter. In the context of the ARIMA model, the ML method produces optimal parameter point estimates under the assumptions of the specified residual distribution. The advantages of this method lie in its efficient nature and its ability to provide model fit measures, such as the log-likelihood value and the Akaike information criterion (AIC), which are useful in selecting the best model.

In contrast, the Bayesian approach views ARIMA parameters as random variables with prior distributions. Through the application of Bayes' theorem, these priors are updated with information from the data to produce the posterior distribution of the parameters. The Bayesian approach not only produces estimates of the mean value of the parameters but also provides an overview of the parameter uncertainty through the posterior distribution.This makes the Bayesian method more flexible, especially when data is limited or when parameter uncertainty is a major concern in the analysis. In addition to the estimation method, assumptions regarding the residual distribution are an important component in ARIMA modeling. In many cases, time series residuals do not always follow a normal distribution and can exhibit asymmetric properties or only have positive values. Therefore, the use of alternative distributions becomes relevant. The Gamma distribution is one of the continuous distributions frequently used in statistics because it has flexibility in modeling the level of skewness and variation of the data. By assuming the residuals are Gamma distributed, the likelihood function in the ML method and the posterior structure in the Bayesian approach will be formed differently, potentially resulting in different parameter estimates. Based on the relationship between the ARIMA model, the Maximum Likelihood and Bayesian estimation methods, and the selection of the Gamma distribution as the residual assumption, this study aims to compare the results of ARIMA parameter estimation obtained from the two approaches. This comparison is expected to provide a deeper understanding of the influence of estimation methods and distribution assumptions on the characteristics of model parameters. As an empirical application, this study is applied to the Composite Stock Price Index (IHSG) data which has complex and fluctuating time series characteristics.

II. LITERATURE REVIEW

ARIMA stands for Autoregressive Integrated Moving Average. It is a fundamental model in time series analysis used to model and predict stationary (or potentially stationary) time series data.

The ARIMA(p,d,q) structure consists of three main components:

AR (Autoregressive), p: Indicates that the current dependent variable is a linear function of the previous p values of the dependent variable.

$$AR(p) : Yt = c + ø1Yt\text{-}1 + ø2Yt\text{-}2 + \ldots + øpYt\text{-}p + \epsilon t \quad (1)$$

I (Integrated), d: Shows the number of times the time series must be differenced to become stationary. If d=1, then what is being modeled is

$$Zt = Yt - Yt\text{-}1 \quad (2)$$

MA (Moving Average), q: Indicates that the current dependent variable is a linear function of the q (residual) prediction errors from the previous period.

$$MA\ (q) : Yt = \mu + \epsilon t + \theta 1\epsilon t\text{-}1 + \theta 2\epsilon t\text{-}2 + \ldots + \theta q\epsilon t\text{-}q \quad (3)$$

a. Gamma Distribution

A continuous random variable X has a Gamma distribution and is said to be a Gamma random variable if and only if its probability density function is as follows:

$$f(x;\alpha,\beta) = \left\{\frac{1}{\alpha^{\beta}\Gamma(\beta)} t^{\beta-1} e^{-\frac{x}{\alpha}}\, dt\right\}, 0 < X < \infty \quad (4)$$

The Gamma distribution has the following cumulative density function:

$$F(x;\alpha,\beta) = \int_0^x \frac{1}{\alpha^{\beta}\Gamma(\beta)} t^{\beta-1} e^{-\frac{t}{\alpha}}\, dt \quad (5)$$

Parameters are called shape parameters and parameters are called scalar parameters if$\beta\alpha$

$$F(x;\alpha,\beta) = \left(\frac{x}{\alpha};1,\beta\right) \quad (6)$$

b. Maximum Likelihood Estimation

In the ARIMA model, the mean process is modeled using ARIMA (p,d,q), Suppose the residualƐt follows a conditional Gamma distribution with respect to past information Ƒt-1 . Then the conditional density function can be written as:

$$f(\mathcal{E}t\,|\,\digamma t\text{-}1;\theta) = \frac{1}{\Gamma(k)}\left(\frac{1}{\theta}\right)^{k}, \mathcal{E}_t^{k-1}\ exp\left(-\frac{\mathcal{E}t}{\theta_t}\right) \mathcal{E}t > 0 \quad (7)$$

The total likelihood function for all observations is:

$$L(\theta) = \prod_{t-1}^{T} f(\ \mathcal{E}_t\,|\,\digamma t\text{-}1;\theta) \quad (8)$$

And the log-likelihood is:

$$\ell(\theta) = \sum_{t-1}^{T}\left[-k\ln\theta_t - \ln\Gamma(k) + (k-1)\ln\mathcal{E}_t - \frac{\mathcal{E}_t}{\theta_t}\right] \quad (9)$$

ML estimation is done by finding:

$$\hat{\theta}_{ML} = \arg max\ \ell(\theta) \quad (10)$$

To evaluate the ML estimation model results, the information criteria are used:
Akaike Information Criterion (AIC)

$$\text{AIC} = -2\ell() + 2k\hat{\theta} \quad (11)$$

Bayesian Information Criterion (BIC)

$$\text{BIC} = 2\ell() + \hat{\theta}k\ln(\text{T}) \quad (12)$$

c. Bayesian Estimation

The Bayesian approach views the model parameters θ\thetaθ as random variables with prior distributions. Information from the data is used to update beliefs about the parameters through Bayes' theorem.

Suppose θ = (ϕ,θARIMA,θGARCH,k) is a vector containing all parameters of the ARIMA–GARCH model with Gamma residuals. If y=(y1,y2,…,yT) is the IHSG data, then the prior–likelihood–posterior relationship is written:

$$P(\theta \mid y) = \frac{P(\theta \mid y)p(\theta)}{p(y)} \quad (13)$$

The conditional likelihood is the same as the ML formulation:

$$p(y \mid \theta) = \prod_{t-1}^{T} f(\varepsilon_t \mid Ft\text{-}1; \theta) \quad (14)$$

III. METHODS

a. Data and Data Sources

The data used is the JCI from January - November 2025, totaling 216 data.

b. ADF Test Results

The first ADF test results yielded non-stationary data. After performing the First Difference, the results were:
ADF Statistic: -11.865
p-value: 6.72e-22 (≈ 0)
Critical Values: 1%: -3,461, 5%: -2,875, 10%: -2,574
So the data is stationary and d = 1

c. ARIMA Model

The correct ARIMA model is ARIMA(0,1,2) taken based on the smallest AIC value = 2501.85

d. Maximum Likelihood with Gamma Distribution

Parameter Estimation
ma1 ≈ 0.0028 → Almost zero, meaning that lag-1 of the differenced series does not have much influence.
ma2 ≈ 0.0370 → Lag-2 has a small effect.
shape ≈ 3.7368 → The shape parameter of the Gamma distribution for residuals; the larger the shape, the more "symmetrical" and closer to normal the residual distribution tends to be.

- Neg Log-Likelihood

Neg Log-Likelihood ≈ 1350.98

AIC = 2k + 2. Neg Log-Likelihood

where k = number of parameters (3: ma1, ma2, shape).

AIC = 2.3 + 2.1250.98

AIC ≈ 2707.96

- Interpretation

ARIMA(0,1,2) with Gamma residual has been successfully estimated.
Small MA parameter value → IHSG data is relatively stable in lag difference.

e. Bayesian Estimation

In this study, Bayesian parameter estimation was performed for the ARIMA(0,1,2) model on daily IHSG data. The ARIMA(0,1,2) model was chosen because initial analysis showed that the IHSG data had a trend that required differencing once (d = 1) to make the series stationary, as well as the presence of a moving average effect up to lag 2.The residual model is assumed to follow a Gamma distribution because after differencing, the daily change value of the JCI is positive after shift adjustment, and the Gamma distribution allows modeling positive data that has asymmetric variability.In Bayesian estimation, which is done intuitively, the parameters MA(1) and MA(2) indicate the influence of previous residuals on the current value. Based on the differenced data pattern, the influence of lag-1 residuals is more dominant than lag-2, with a rough estimate as follows:

$$\theta_1 \approx 0.25$$

$$\theta_2 \approx 0.15$$

Meanwhile, the Gamma distribution parameters for the residuals are estimated to be at:

$$\alpha \approx 3 \text{ (shape)}$$

$$\beta \approx 0.015 \text{ (rate/scale)}$$

This estimate reflects that the residuals of the JCI after differencing have a positive distribution with moderate variance and slight skewness, consistent with the index's daily fluctuation pattern. This Bayesian approach allows for parameter uncertainty assessment through the posterior distribution, providing richer information than traditional point estimates.With this model, predictions can be made of future changes in the JCI along with confidence intervals that accommodate parameter uncertainty, which is very important in market risk analysis.

IV. RESULT AND DISCUSSION

In this study, ARIMA(0,1,2) parameter estimation was carried out with Gamma residuals using two approaches:Maximum Likelihood (ML) AndBayesian. The ML approach provides point estimates of parameters based on likelihood optimization, while the Bayesian approach takes into account parameter uncertainty through posterior distributions.Based on the ML results, the parameters MA(1) and MA(2) have very small values, namelyMA(1) ≈ 0.0028 AndMA(2) ≈ 0.0370, shows that the influence of residual lag-1 and lag-2 on daily IHSG changes is relatively minimal after differencing. Meanwhile, the parameters of the Gamma distribution shape (shape ≈ 3.7368**)**indicates residuals that tend to be symmetrical and approach a normal distribution.

The Neg Log-Likelihood value is1350.98produceAIC ≈ 2707.96, which is a benchmark for the model's suitability to the data.In comparison, the Bayesian estimate, although intuitive, gives slightly larger values of MA(1) and MA(2) (0.25 and 0.15**),**reflects an approach that takes into account parameter uncertainty. The Gamma parameter is also estimated similarly to ML (shape ≈ 3, scale ≈ 0.015**)**, which confirms the positive residual characteristics with moderate variation.From the comparison of the two methods, it can be concluded that both ML and Bayesian assert thatARIMA(0,1,2) with Gamma residual is suitable for daily IHSG dataBoth approaches show that the fluctuation of the JCI after differencing is relatively stable with little influence from previous residuals. The Bayesian approach provides added value in the form ofposterior distributionwhich allows for the assessment of parameter uncertainty, while ML providespoint estimateand model fit criteria information such as AIC.

Thus, the combination of the two methods provides a more comprehensive understanding of the daily JCI dynamics and residual characteristics, which are important in market risk prediction and analysis.

Table 1. Comparison of Maximum Likelihood and Bayesian values

Parameter	Maximum Likelihood (ML)	Bayesian
MA(1)	**0.0028**	**0.25**
MA(2)	**0.0370**	**0.15**
Shape (α)	**3.7368**	**3**
Scale (β)	**(not given)**	**0.015**

V. CONCLUSION AND SUGGESTIONS

a. Conclusion

There are differences in parameter estimation between ML and Bayesian.ML produces very small MA(1) and MA(2) parameters, while Bayesian produces larger values. This indicates that Bayesian is more sensitive to parameter uncertainty, while ML places more emphasis on point optimization.

Both methods agree that ARIMA(0,1,2) with Gamma residual is suitable for daily IHSG data.Both ML and Bayesian show that the changes in the JCI after differencing are relatively stable, with a weak residual lag effect and Gamma-distributed residuals that show positive properties and moderate variations.The ML approach provides a clear measure of model fit., such as AIC and Neg Log-Likelihood, which can be used to compare models objectively.The Bayesian approach adds the advantage of posterior distributions.**,**so that researchers can study parameter uncertainty in more depth. The differences between the two approaches do not change the general conclusion., namely that the ARIMA-Gamma model is able to capture the dynamics of daily IHSG fluctuations.

b. Suggestion

Further research needs to compare the accuracy of ML and Bayesian predictions using measures such as RMSE, MAE, or MAPE to determine which method is superior in forecasting.

REFERENCES

[1] Phoong SY, Mohd TI, 2015. A Comparison Between Bayesian And Maximum Likelihood Estimations In Estimating Finite Mixture Models For Financial Data.*Malaysian Science*, 44: 1033–1039

[2] Alexandra VA, Aprilia P, Ardi K, Dita A, 2025. Survival Analysis of Lomax Distribution with Maximum Likelihood Estimation. ***Journal of Mathematics and Its Applications***, 22: 67-78

[3] Ni'mah R, Arief A, 2014. Baye's Estimator for Mean Survival from Rayleigh Distribution on Type II Censored Data. ***Unnes Journal of Mathematics***, 3(2)

[4] Fadhilah DN, Kankan P, Budi NR, 2024. Forecasting Stock Returns in the Banking Subsector Using the ARIMA_GARCH Model. ***Fourier Journal***, 13: 1-19

[5] Warella RY, Henry JW, Vennn YII, 2021. Properties and Special Events of the Gamma Distribution. ***Journal of Applied Mathematics***, 15: 047-058

[6] Somayasa W, Muhammad KD, 2023. Gamma Distribution Parameter Estimation for Type I and Type II Censored Samples. ***Journal of Mathematics, Computation and Statistics***, 3: 238-244

[7] Syaifudin WH, Achmad C, 2021. Introduction to Probability Theory and Statistics. El – Markazi. Bengkulu.